Quasi-coherent state of pions in the nucleon

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Abstract

Making use of the quasi-coherent state developed by Eriksson *et al.*, we can find a nucleon solution accompanied by the pion field with trivial topology. We compare our approach with other related works, and examine a coherent state description of pions in the baryon structure. Our solution suggests a kind of nucleon resonance due to the topological change of pion field without the usual quark excitation.

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I. INTRODUCTION

The pion has been accepted as one of the fundamental degrees of freedom in the hadron physics. The effective theories for the baryon structure, in which the broken chiral symmetry is taken into consideration, provide strong coupling and nonlinearity to the pion. Many efforts have been made to deal with these nonperturbative features in order to reveal a pion contribution to the baryon structure.

A possible approach is the coherent state description for the pion field. This approach is traced back to the intermediate coupling approximation developed and succeeded in several applications in the particle and condensed matter physics [1, 2]. The idea of the coherent state has been already exploited in the hedgehog ansatz [3, 4] and the coherent pair approximation (CPA) [5, 6]. The hedgehog ansatz especially is capable of reproducing ground state properties of the nucleon [4, 7].

However the coherent state has not been fully examined yet. The hedgehog ansatz is not a necessary condition for the pion field interacting with other constituents, although this ansatz is responsible for assigning the baryon number to the Skyrmion [3, 8]. As for the CPA, its theoretical foundation is not clear since the spin-isospin symmetry is respected in an intuitive manner.

Furthermore, a problem concerning the isospin symmetry of the pion is inherent in the coherent state description. The two assumptions stated above are closely related with this problem. The customary method of making the coherent state breaks the isospin symmetry. Eriksson *et al.* managed to obtain the isospin-conserving coherent state for the pion field, which is called the quasi-coherent state (QCS) [9], by using the Peierls-Yoccoz (PY) projection operator [10].

Now we try to utilize the QCS for studying the pion properties in a baryon without holding on the hedgehog ansatz or the CPA. In this paper we calculate the ground state mass of the nucleon in the linear sigma model, employing the QCS for the pion field. Throughout our discussion we compare our approach with the works related with the coherent state, i.e. the hedgehog ansatz [4] and the CPA [6, 11].

In Sec. II we introduce the standard coherent state for the pion field and construct the QCS in a general form. In Sec. III, after a brief comment on the linear sigma model for the nucleon, we make a nucleon state from the quark, sigma, and pion states. Sec. IV is devoted

to explaining the variational calculation for the nucleon mass. There we comment on our assumption for the pion distribution function. In Sec. V, we compare the nucleon mass calculated by using the QCS with those obtained by the CPA and the hedgehog ansatz. We carefully discuss similarity and difference among these models, taking notice on the pion distribution in each model. Finally summary and our future perspectives are given in Sec. VI.

II. QUASI-COHERENT STATE

The standard coherent state $|\Pi\rangle$ for the pion field is defined by

$$\hat{a}_i(\mathbf{k})|\Pi\rangle = f_i(\mathbf{k})|\Pi\rangle \ , \tag{1}$$

where $\hat{a}_i(\mathbf{k})$ ($\hat{a}_i^\dagger(\mathbf{k})$) is the annihilation (creation) operator with the momentum $\mathbf{k}$ and the ith isospin component, satisfying the commutation relation $\left[\hat{a}_i(\mathbf{k}), \hat{a}_j^\dagger(\mathbf{k}')\right] = \delta_{ij}\delta^3(\mathbf{k}-\mathbf{k}')$. The complex function $f_i(\mathbf{k})$ determines the pion distribution. Explicitly $|\Pi\rangle$ is given by (apart from the normalization factor)

$$|\Pi\rangle = \exp\left[\int d^3k \boldsymbol{f}(\mathbf{k})\cdot\hat{\boldsymbol{a}}^\dagger(\mathbf{k})\right]|0\rangle \ , \tag{2}$$

where the dot represents the scalar product in the isospace.

For our later use, we introduce the spherical representation of $\hat{a}_i^\dagger(\mathbf{k})$

$$\hat{a}_{\mu lm}^\dagger(k) = (-i)^l \int d\hat{k} Y_{lm}(\hat{k})\hat{a}_\mu^\dagger(\mathbf{k}) \ , \tag{3}$$

where $\hat{k}$ denotes the direction of $\mathbf{k}$, and μ takes ±1 or 0 with the definitions

$$\hat{a}_\pm^\dagger(\mathbf{k}) = \mp\frac{1}{\sqrt{2}}\left[\hat{a}_1^\dagger(\mathbf{k}) \pm i\hat{a}_2^\dagger(\mathbf{k})\right] \ , \ \ \hat{a}_0^\dagger(\mathbf{k}) = \hat{a}_3^\dagger(\mathbf{k}) \ . \tag{4}$$

By expanding the distribution function as $f_\mu(\mathbf{k}) = \sum(-i)^l f_{\mu lm}(k)Y_{lm}(\hat{k})$, Eq. (2) becomes

$$|\Pi\rangle = \exp\left[\int dk \sum_{\mu lm}(-)^\mu f_{-\mu lm}(k)\hat{a}_{\mu lm}^\dagger(k)\right]|0\rangle \ . \tag{5}$$

Because $|\Pi\rangle$ is not an eigenstate both of the spin and isospin operators, we extract an eigenstate with the isospin (T,μ) and the angular momentum (L,M) from $|\Pi\rangle$ by using the Peierls-Yoccoz (PY) projection [10],

$$|\boldsymbol{f};T\mu\nu;LMK\rangle \equiv P_{\mu\nu}^T P_{MK}^L|\Pi\rangle \ . \tag{6}$$

$P^T_{\mu\nu}$ is the PY operator for the isospin,

$$P^T_{\mu\nu} = \int dg \; D^{T*}_{\mu\nu}(g)\hat{R}(g) \;, \tag{7}$$

where $D^{T*}_{\mu\nu}(g)$ is the rotation matrix [12], and g represents the Euler angle in the isospace. The measure is defined as $\int \mathrm{d}g = 1$. In Eq. (7), the factor $2T+1$ is dropped from the usual definition for brevity. Similarly P^L_{MK} is for the angular momentum,

$$P^L_{MK} = \int dh D^{L*}_{MK}(h) R(h) \;. \tag{8}$$

The Euler angle in the coordinate space is represented by h. Reference [9] first introduced Eq. (7) and called it the quasi-coherent state.

Note that the indices ν and K are redundant in Eq. (6) because there is no 'intrinsic axis' for the pion distribution. In fact the states (6) are orthogonal with respect to the indices (T,μ) and (L,M), but not to ν and K. We take a linear combination [13]

$$|\boldsymbol{f}; T\mu; LM\rangle = \sum_{\nu K} C_{\nu K} |\boldsymbol{f}; T\mu\nu; LMK\rangle \;, \tag{9}$$

and we consider Eq. (9) as a pion state in our calculation.

III. HAMILTONIAN AND NUCLEON STATE

We consider the static Hamiltonian corresponding to the linear sigma model,

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 \;. \tag{10}$$

$\mathcal{H}_0$ is

$$\mathcal{H}_0 = \hat{\psi}(\mathbf{r})^\dagger(-i\boldsymbol{\alpha}\cdot\nabla)\hat{\psi}(\mathbf{r}) + \frac{1}{2}\left[\hat{P}_\sigma(\mathbf{r})^2 + \nabla\hat{\sigma}(\mathbf{r})^2\right] + \frac{1}{2}\left[\hat{\boldsymbol{P}}_\pi(\mathbf{r})^2 + \nabla\hat{\boldsymbol{\pi}}(\mathbf{r})^2\right] \;, \tag{11}$$

where $\hat{\psi}$ is the massless quark field, and $\hat{\sigma}$ ($\hat{P}_\sigma$) and $\hat{\pi}$ ($\hat{\boldsymbol{P}}_\pi$) are the sigma and pion fields (their conjugate fields), respectively. The meson-quark interaction and meson self-interaction are included in

$$\begin{aligned}\mathcal{H}_1 \;=\; & G\bar{\hat{\psi}}(\mathbf{r})\left[\hat{\sigma}(\mathbf{r}) + i\gamma_5\boldsymbol{\tau}\cdot\hat{\boldsymbol{\pi}}(\mathbf{r})\right]\hat{\psi}(\mathbf{r}) \\ & + \frac{\lambda^2}{4}\left[\hat{\sigma}(\mathbf{r})^2 + \hat{\boldsymbol{\pi}}(\mathbf{r})^2 - \nu^2\right]^2 - m_\pi^2 f_\pi\left[\hat{\sigma}(\mathbf{r}) - f_\pi\right] - \frac{m_\pi^4}{4\lambda^2} \;,\end{aligned} \tag{12}$$

where G is the coupling constant, m_π the pion mass, and f_π the pion decay constant. The parameter ν and the self-interaction strength λ is related to m_π, f_π, and the sigma mass m_σ as $\nu^2 = f_\pi^2 - m_\pi^2/\lambda^2$, $\lambda^2 = (m_\sigma^2 - m_\pi^2)/(2f_\pi^2)$.

We consider the ground state baryon in this work, and assume that all quarks are in the lowest s-wave. We expand the quark field as

$$\hat{\psi}(\mathbf{r}) = \sum_{\mu m} \left(\langle \mathbf{r} | \mu m \rangle \hat{d}_{\mu m} + \langle \mathbf{r} | \mu m \rangle^* \hat{d}^\dagger_{\mu m} \right) \ , \tag{13}$$

where $\hat{d}_{\mu m}$ annihilates the quark with the isospin μ and the spin m. The quark spinor is written as

$$\langle \mathbf{r} | \mu m \rangle = \begin{pmatrix} u(r) \\ iv(r) \boldsymbol{\sigma} \cdot \hat{\boldsymbol{r}} \end{pmatrix} \chi_m \zeta_\mu \ , \tag{14}$$

where χ_m is the two-spinor, and ζ_μ the isospinor.

Since the quark excitation is not considered, the sigma meson always remains in the s-wave through the scalar interaction in $\mathcal{H}_1$. And only the p-wave pion can interact with the quark through the pseudoscalar interaction. The meson fields are expanded in the spherical representation as

$$\hat{\sigma}(\mathbf{r}) \ = \ \int_0^\infty \frac{\mathrm{d}k k^2}{\sqrt{\pi \omega_\sigma}} j_0(kr) \left[\hat{c}(k) + \hat{c}^\dagger(k) \right] Y^*_{00}(\hat{r}) \ , \tag{15}$$

$$\hat{\pi}_\mu(\mathbf{r}) \ = \ \int_0^\infty \frac{\mathrm{d}k k^2}{\sqrt{\pi \omega_\pi}} \sum_m j_1(kr) \left[(-)^{\mu+m} \hat{a}_{-\mu,-m}(k) + \hat{a}^\dagger_{\mu m}(k) \right] Y^*_{1m}(\hat{r}) \ , \tag{16}$$

where $\omega_\sigma = \sqrt{k^2 + m_\sigma^2}$, $\omega_\pi = \sqrt{k^2 + m_\pi^2}$, and $j_l(kr)$ is the spherical Bessel function. The annihilation operator for the s-wave sigma field is denoted by $\hat{c}(k)$, and $\hat{a}_{\mu 1 m}(k)$ is simply written as $\hat{a}_{\mu m}(k)$.

We consider that a nucleon state is composed of quark, pion, and sigma states as in Refs. [4, 6]. Following the usual prescription in the constituent quark model [14], we make a three-quark state with the spin-isospin 1/2 denoted by $|3\mathrm{q}; N\rangle$ and that with 3/2 by $|3\mathrm{q}; \Delta\rangle$.

The standard coherent state description is used for a sigma meson state,

$$|\Sigma\rangle = \exp \left\{ \int_0^\infty dk k^2 \left[\eta(k)^* \hat{c}(k) - \eta(k) \hat{c}^\dagger(k) \right] \right\} |0\rangle \ , \tag{17}$$

which satisfies the standard definition, $\hat{c}(k)|\Sigma\rangle = \eta(k)|\Sigma\rangle$. The complex function $\eta(k)$ represents the momentum distribution.

Employing the QCS for the pion state explained in the last section, we write down the nucleon state as a linear combination with the coefficients α, β, and γ,

$$|N\rangle \;=\; \Big(\alpha|3\mathrm{q};N\rangle|\boldsymbol{f};00\rangle + \beta\,[|3\mathrm{q};N\rangle\otimes|\boldsymbol{f};11\rangle]^{\frac{1}{2}\frac{1}{2}} + \gamma\,[|3\mathrm{q};\Delta\rangle\otimes|\boldsymbol{f};11\rangle]^{\frac{1}{2}\frac{1}{2}}\Big)\,|\Sigma\rangle\ , \quad (18)$$

where $|\boldsymbol{f};00\rangle$ ($|\boldsymbol{f};11\rangle$) means the QCS with $T=L=0$ ($T=L=1$). We obtain the delta state $|\Delta\rangle$ with the spin-isospin 3/2 by interchanging $|3\mathrm{q};N\rangle$ and $|3\mathrm{q};\Delta\rangle$ in Eq. (18).

IV. CALCULATION

We calculate the expectation value of $H=\int d^3r\mathcal{H}$ by the variational method,

$$\delta\left\{\langle N|H|N\rangle - E\langle N|N\rangle - 12\pi\epsilon\int_0^\infty drr^2\left[u(r)^2+v(r)^2\right]\right\}=0\ , \qquad (19)$$

where the Lagrange multipliers E and ϵ are introduced to normalize $|N\rangle$ and the quark wave function, and E corresponds to the nucleon mass. We notice that the QCS (9) is not normalized and its norm depends on $f_\mu(\mathbf{k})$. The variation is taken with respect to the coefficients α, β, and γ, and also to the quark and meson fields. Then we obtain the energy eigenvalue equation for the coefficients and the differential equations for the fields.

Here we consider the pion distribution $f_\mu(\mathbf{k})$. If there is no apparent correlation between the isospin and coordinate spaces, we can write the p-wave component of $f_\mu(\mathbf{k})$ as $-if_{\mu0}(k)Y_{10}(\hat{k})$, taking into account the axial symmetry of $\mathcal{H}$. Further we assume that all the three components have the same momentum dependence for simplicity: $f_{\mu0}(k)=f_\mu\xi(k)$, where f_μ is a constant vector in the isospace and $\xi(k)$ is a real function of k. This simplest form we choose here should be compared with other choices such as the hedgehog form. We discuss this point further in the next section.

We do not take the variation with respect to $C_{\nu K}$ introduced in the QCS (9) because these coefficients are related with f_μ. Let us consider, for example, the matrix element of the pion kinetic energy $H_\pi=\int \mathrm{d}^3k\omega_\pi\hat{\boldsymbol{a}}^\dagger(\mathbf{k})\cdot\hat{\boldsymbol{a}}(\mathbf{k})$ between the QCS (9),

$$\begin{aligned}&\langle\boldsymbol{f};T\mu;LM|H_\pi|\boldsymbol{f};T\mu;LM\rangle\\ &=\sum_{\nu'\nu\lambda'\lambda}C^*_{\nu'0}C_{\nu0}f^*_{\lambda'}f_\lambda\int dgdhD^{1*}_{\nu'\nu}(g)D^{1*}_{\lambda'\lambda}(g)D^{1*}_{00}(h)D^{1}_{00}(h)F(g,h)\int_0^\infty dkk^2\omega_\pi\xi(k)^2\ ,\end{aligned}\qquad (20)$$

where only $C_{\nu K}$ with $K=0$ appears because of the axial symmetry of $f_\mu(\mathbf{k})$. The function $F(g,h)$ is defined as

$$F(g,h) = \exp\left[s\ D^1_{00}(h) \sum_{\rho'\rho} D^1_{\rho'\rho}(g)\ f_{\rho'} f^*_\rho \right] \ , \tag{21}$$

where the norm integral is given by

$$s = \int_0^\infty dk k^2 \xi(k)^2 \ . \tag{22}$$

We notice that s is not a pripri normalized as $s=1$. Here we consider the vector f'_μ which is related with f_μ through the Euler angle g' as $f_\mu = \sum_\nu D^{1*}_{\mu\nu}(g') f'_\nu$. With this f'_μ, Eq. (20) becomes as

$$\begin{aligned}(20) \ = \ & \sum_{\nu'\nu\lambda'\lambda} C'^*_{\nu'0} C'_{\nu 0} f'^*_{\lambda'} f'_\lambda \\ & \times \int dg dh D^{1*}_{\nu'\nu}(g) D^{1*}_{\lambda'\lambda}(g) D^{1*}_{00}(h) D^1_{00}(h) F'(g,h) \int_0^\infty dk k^2 \omega_\pi \xi(k)^2 \ ,\end{aligned} \tag{23}$$

where $C'_{\nu 0} = \sum_\sigma C_{\sigma 0} D^1_{\sigma\nu}(g')$, and $F'(g,h)$ is obtained from Eq. (21) by exchanging f_μ with f'_μ. Equations (20) and (23) show that the matrix element is invariant with respect to the simultaneous rotation of f_μ and $C_{\nu 0}$ in the isospace. Because this is also the case for all other matrix elements, we take the special value $C_{\nu 0} = (0,1,0)$ and vary the direction of f_μ.

We write the matrix elements of H following Refs. [6, 11],

$$\langle N|H|N\rangle = 4\pi \int_0^\infty dr r^2 \left[\alpha^2 E_{\alpha\alpha}(r) + \beta^2 E_{\beta\beta}(r) + \gamma^2 E_{\gamma\gamma}(r) + 2\alpha\beta E_{\alpha\beta}(r) + 2\alpha\gamma E_{\alpha\gamma}(r) \right] , \tag{24}$$

where $E_{\alpha\beta} = E_{\beta\alpha}$, $E_{\alpha\gamma} = E_{\gamma\alpha}$, and $E_{\beta\gamma} = E_{\gamma\beta} = 0$. The energy densities E_{ij} are expressed in terms of the quark fields and the meson fields $\sigma(r), \phi(r)$ (and $\phi_p(r)$) defined as

$$\sigma(r) \ = \ 2 \int_0^\infty \frac{dk k^2}{\sqrt{\omega_\sigma}} j_0(kr) \left[\eta(k) + \eta^*(k) \right] \ , \tag{25}$$

$$\phi(r) \ = \ \frac{1}{2\pi} \int_0^\infty \frac{dk k^2}{\sqrt{\omega_\pi}} j_1(kr) \xi(k) \ , \tag{26}$$

$$\phi_p(r) \ = \ \frac{2}{\pi} \int_0^\infty dr' r'^2 \int_0^\infty dk k^2 \omega_\pi j_1(kr) j_1(kr')\ \phi(r') \ . \tag{27}$$

The diagonal parts are

$$E_{ii}(r) = E_0(r) n^{ii}_0 + 2\phi_p^2\ n^{ii}_1 + \lambda^2 \left(\sigma^2 - f_\pi^2 \right) \phi^2\ n^{ii}_2 + \frac{\lambda^2}{4} \phi^4\ n^{ii}_3 \ , \tag{28}$$

where $i = \alpha, \beta, \gamma$ and

$$
\begin{aligned}
E_0(r) \ &= \ 3\left(2u\frac{dv}{dr} + 4\frac{1}{r^2}uv\right) + \frac{1}{2}\left(\frac{d\sigma}{dr}\right)^2 + 3g\sigma\left(u^2 - v^2\right) \\
&\quad + \frac{\lambda^2}{4}\left(\sigma^2 - f_\pi^2\right)^2 + \frac{m_\pi^2}{2}\left(\sigma^2 - f_\pi^2\right) - m_\pi^2 f_\pi\left(\sigma - f_\pi\right) \ , \qquad (29)
\end{aligned}
$$

where n_k^{ii} $(k = 0, 1, 2, 3)$ represent the integrals with respect to the Euler angles g, h. The off-diagonal parts are

$$E_{\alpha j}(r) = -g\ C_{\alpha j}\ uv\phi\ n^{\alpha j} \ , \qquad (30)$$

where $j = \beta, \gamma$, and $C_{\alpha\beta} = 10/\sqrt{3}$, $C_{\alpha\gamma} = 8\sqrt{2/3}$. The explicit forms of n_k^{ii} and $n^{\alpha j}$ are summarized in Appendix.

The norm of $|N\rangle$ is expressed in terms of n_0^{ii} as

$$\langle N|N\rangle = \alpha^2 n_0^{\alpha\alpha} + (\beta^2 + \gamma^2) n_0^{\beta\beta} \ . \qquad (31)$$

We determine the mixing coefficients α, β, and γ so that $\langle N|N\rangle = 1$.

The differential equations for the quark and meson fields are

$$
\begin{aligned}
\frac{du}{dr} \ &= \ -\left(G\sigma + \epsilon\right)v - \frac{2}{3}G\alpha\delta_N\ u\phi\ n^{\alpha\beta} \ , \qquad (32) \\
\frac{dv}{dr} \ &= \ -\frac{2}{r}v - \left(G\sigma - \epsilon\right)u + \frac{2}{3}G\alpha\delta_N\ v\phi\ n^{\alpha\beta} \ , \qquad (33) \\
\frac{d^2\sigma}{dr} \ &= \ -\frac{2}{r}\frac{d\sigma}{dr} + 3G\left(u^2 - v^2\right) + \lambda^2\left(\sigma^2 - f_\pi^2\right)\sigma + m_\pi^2\left(\sigma - f_\pi\right) \\
&\quad +2\lambda^2\phi^2\sigma\left(\boldsymbol{f}\cdot\boldsymbol{f} + \frac{1}{9}N_\pi\right) \ , \qquad (34) \\
\frac{d^2\phi}{dr^2} \ &= \ -\frac{2}{r}\frac{d\phi}{dr} + \frac{2}{r^2}\phi + m_\pi^2\phi \\
&\quad +\frac{\lambda^2}{2}\left(\sigma^2 - f_\pi^2\right)\phi\left(1 + \boldsymbol{f}\cdot\boldsymbol{f}\frac{s}{N_\pi}\right) + \frac{\lambda^2}{4}\frac{2}{N_\pi}\left[\alpha^2 n_3^{\alpha\alpha} + (\beta^2 + \gamma^2) n_3^{\beta\beta}\right]\phi^3 \\
&\quad -G\ \alpha\ \delta_N\ \frac{s}{N_\pi}\ n^{\alpha\beta}\ uv - E\ \phi_p + \frac{1}{N_\pi}\ \phi_p\ \vartheta(\alpha, \beta, \gamma, u, v, \sigma, \phi, s, f_\mu) \ , \qquad (35)
\end{aligned}
$$

where $\delta_N = (5\beta + 4\sqrt{2}\gamma)/\sqrt{3}$, and $N_\pi = \alpha^2 n_1^{\alpha\alpha} + (\beta^2 + \gamma^2) n_1^{\beta\beta}$ is the expectation value of the pion number operator. The explicit form of ϑ is not exhibited here because it is lengthy but its derivation is straightforward.

The boundary conditions are

$$\left.\frac{d\sigma}{dr}\right|_{r=0} = 0, \quad v(0) = 0, \quad \phi(0) = 0 \ , \qquad (36)$$

and for $r \to \infty$

$$
\begin{aligned}
\left[r(g^2 f_\pi^2 - \epsilon^2)^{1/2} + 1\right] u - r(g f_\pi + \epsilon) v &= 0 \ , \\
(2 + 2m_\pi r + m_\pi^2 r^2)\phi + (r + m_\pi r^2)\frac{d\phi}{dr} &= 0 \ , \\
(1 + m_\sigma r)\,(\sigma - f_\pi) + r\frac{d\sigma}{dr} &= 0 \ .
\end{aligned}
\tag{37}
$$

We calculate the nucleon mass by solving the differential equations (32)-(35) with the boundary conditions (36), (37) by the iteration procedure.

Before finishing this section, we comment on the CPA for the pion field [6, 11]. We found some errors in Refs. [6, 11]. Their treatment of the coherence parameter x as an independent variable is misunderstanding. Since the dependence on other quantities is dismissed, the value of x is not correctly determined in their calculation of the nucleon mass. In order to compare our result with that of the CPA, we calculate the nucleon mass in the CPA, too.

V. DISCUSSION

a. Quasi-coherent state and Coherent pair state

We calculate the nucleon and delta masses. The pion mass and the decay constant are fixed to the observed values: $m_\pi = 140$ MeV, $f_\pi = 93$ MeV. The free parameters in our model are the pion-quark coupling constant G and the sigma mass m_σ. We choose the typical values for G and m_σ in order to compare our results directly with those calculated by using the CPA [6, 11] and the hedgehog ansatz [4].

Using the parameter set $G = 5$, $m_\sigma = 700$ MeV taken from Ref. [11], we find a self-consistent solution in our model with the QCS for the pion field. The quark and meson fields ($u(r)$, $v(r)$, $\phi(r)$, and $\sigma(r)$) are exhibited in Fig. 1. The nucleon mass ($E_N^{\rm QCS}$) and the delta mass ($E_\Delta^{\rm CPA}$) become 1113 MeV and 1248 MeV, respectively.

When we employ the CPA for the pion field instead of the QCS, we obtain $E_N^{\rm CPA} = 1093$ MeV and $E_\Delta^{\rm CPA} = 1233$ MeV. Because we correct some errors in Refs. [6, 11], $E_N^{\rm CPA}$ is larger than the value found in these references by about 20 MeV.

As long as we notice $E_N^{\rm QCS} > E_N^{\rm CPA}$, the CPA state looks better than the QCS as a trial function in the variation method. However, this difference is only about 2% of the observed nucleon mass (940 MeV), and this is also the case for the Δ mass. Thus we carefully compare the QCS with the CPA state. We can expect similarity in the structure between these states.

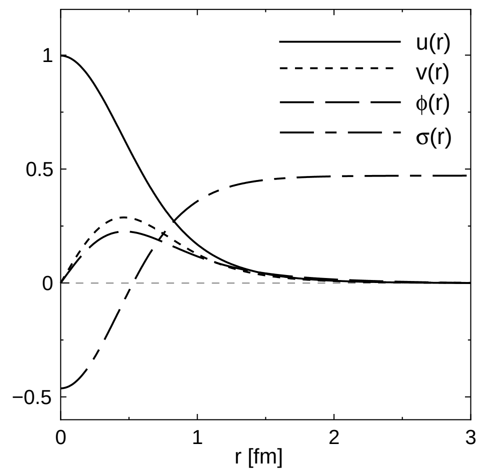

FIG. 1: The quark and meson fields for the nucleon using the parameter set $G = 5$, $m_\sigma = 700$ MeV. The pion field is described by the QCS.

As explained in Sec. II, the QCS is extracted from the standard coherent state by using the PY projector both for the isospin and the angular momentum. We verify a similarity between the CPA state and the QCS by assuming $f_\mu(\mathbf{k}) = f_\mu \xi(\mathbf{k})$ as before. First we consider only the isospin projection in the definition Eq. (6),

$$|\boldsymbol{f}; T\mu\nu\rangle = P^T_{\mu\nu} \exp\left(\boldsymbol{f} \cdot \boldsymbol{b}^\dagger\right)|0\rangle \ , \tag{38}$$

where $\boldsymbol{b}^\dagger$ is defined by

$$b^\dagger_\mu = \int d^3k \xi(\mathbf{k}) a^\dagger_\mu(\mathbf{k}) \ . \tag{39}$$

Making use of the Rayleigh expansion, we rewrite Eq. (38) further as [15]

$$|\boldsymbol{f}; T\mu\nu\rangle = \sqrt{\frac{4\pi}{2T+1}} 2^T |f|^T Y^*_{T\nu}(\hat{f}) \sum_n \frac{4\pi 2^T (n+T)!}{n!(2n+2T+1)!} (\boldsymbol{f}\cdot\boldsymbol{f})^n \left(\boldsymbol{b}^\dagger \cdot \boldsymbol{b}^\dagger\right)^n Y_{T\mu}(\boldsymbol{b}^\dagger)|0\rangle \ . \tag{40}$$

This state satisfies the equation

$$\boldsymbol{b}\cdot\boldsymbol{b}|\boldsymbol{f}; T\mu\nu\rangle = \boldsymbol{f}\cdot\boldsymbol{f} s'^2 |\boldsymbol{f}; T\mu\nu\rangle \ , \tag{41}$$

where $\boldsymbol{b}\cdot\boldsymbol{b} = \sum_\mu (-)^\mu b_\mu b_{-\mu}$ and $s' = \int d^3k \xi(\mathbf{k})^2$. The QCS is reduced to the CPA state, if we put the momentum dependence aside from our consideration.

Next the angular momentum is projected, and the QCS with $T = L = 0$ is explicitly written as

$$|\boldsymbol{f}; 00\rangle \propto \left[1 + \frac{1}{18}(\boldsymbol{f}\cdot\boldsymbol{f})(\boldsymbol{b}^\dagger \cdot \boldsymbol{b}^\dagger) + \cdots\right]|0\rangle \ , \tag{42}$$

where

$$b^{\dagger}_{\mu m} = \int_0^{\infty} \mathrm{d}k k^2 \xi(k) a^{\dagger}_{\mu m}(k) \ , \tag{43}$$

and $\boldsymbol{b}^{\dagger} \cdot \boldsymbol{b}^{\dagger} = \sum (-)^{\mu+m} b^{\dagger}_{\mu m} b^{\dagger}_{-\mu,-m}$. The QCS (42) is no longer the eigenstate of the pion pair $\boldsymbol{b} \cdot \boldsymbol{b}$, and is not equivalent to the CPA state for the pion field in the strict sense.

The first two terms of Eq. (42) exactly agree with those of the expanded form of the CPA pion state [6]. They seem to give the dominant contribution to the nucleon mass in each model since $\boldsymbol{f} \cdot \boldsymbol{f}$ and $\langle \boldsymbol{b}^{\dagger} \cdot \boldsymbol{b}^{\dagger} \rangle \sim 1$ in our calculation. Furthermore the coherence parameter x in Refs. [6, 11] corresponds to our $\boldsymbol{f} \cdot \boldsymbol{f}$. This situation is also true for the pion state with $T = L = 1$. Thus the structure of the CPA state, which is intuitively defined in analogy with the standard coherent state, can be clearly understood on the basis of the QCS. Indeed, no essential difference is exposed between the QCS and the CPA state in our calculation of the nucleon mass.

b. Quasi-coherent state and Hedgehog ansatz

We take $G = 5$ and $m_{\sigma} = 1200$ MeV from Ref. [4], and obtain $E_N^{\mathrm{QCS}} = 1215$ MeV. Employing the hedgehog ansatz for the pion field, Fiolhais *et al.* obtained $E_N^{hh} = 938$ MeV [4], which is lower than our result by about 300 MeV! We will show below that the difference between these two models is in a functional form of the pion distribution.

It is known that the pion field satisfying the hedgehog ansatz is closely related to the coherent state [4]. The expectation value of the field operator between the standard coherent state $|\Pi\rangle$ is written as

$$\langle \Pi | \hat{\pi}_{\mu}(\mathbf{r}) | \Pi \rangle = 2i \int_0^{\infty} \frac{dk \ k^2}{\sqrt{\pi \omega_{\pi}(k)}} \sum_m j_1(kr) \ \mathrm{Im} f_{\mu m}(k) Y_{1m}(\hat{r}) \ , \tag{44}$$

where only the p-wave pion is included as before. If we assume the correlation between the coordinate and isovector spaces, $f_{\mu m}(k) = f(k)\delta_{\mu m}$, Eq. (44) becomes

$$(44) = 2i Y_{1\mu}(\hat{r}) \int_0^{\infty} \frac{dk \ k^2}{\sqrt{\pi \omega_{\pi}(k)}} j_1(kr) \ \mathrm{Im} f(k) \ , \tag{45}$$

and we obtain $\langle \Pi | \pi_{\mu}(\mathbf{r}) | \Pi \rangle = i Y_{1\mu}(\hat{r}) \Phi(r)$ corresponding to the hedgehog ansatz.

A difference is in the topological realization for the pion distribution. The hedgehog ansatz takes a non-trivial configuration for the coordinate-isospin mapping with the winding number 1 in $\pi_3(S_3) = Z$ [3]. On the other hand, the pion distribution in our QCS is

topologically trivial, i.e. the winding number 0. This mathematical difference in geometrical character has significant effect on our physical interpretation of the nucleon structure obtained in the models. The mass difference should not be taken merely as a matter of choice of a trial function in the variational method.

The above discussion shows that a pure imaginary function may be chosen to the pion distribution if the hedgehog ansatz is considered. In our model, however, the pion distribution takes a simple real form. We can show that our equations are independent of the phase of a complex function $f_\mu(\boldsymbol{k})$ as far as the baryon masses are considered. Note that the expectation value of $\hat{\pi}_\mu(\mathbf{r})$ for the QCS is not proportional to $\mathrm{Im} f_{\mu m}(k)$ because of the PY projection on the pion coherent state.

We consider that the 300 MeV-difference in the nucleon mass is related with the topological problem. This observation leads us to the following interpretation on our solution. Insofar as the ground state properties are concerned, we can accept that the hedgehog pion may be suitable for the nucleon. The pion distribution in the QCS is topologically distinguished from that in the hedgehog ansatz, and our solution may correspond to the excited state of the nucleon. This excitation is caused by the change in the pion configuration, which is completely different from the usual mechanism of baryon excitation in a constituent quark model. We know that some kinds of the nucleon resonances, such as the Roper resonance, are not fully explained by the quark excitation. We need to solve this problem, for example, by introducing new degrees of freedom other than the constituent quarks. The novel structure of our solution may be taken as one of the possible mechanisms for the baryon excitation.

VI. SUMMARY AND PERSPECTIVES

We have calculated the nucleon mass in the linear sigma model describing the pion field by using the standard coherent state. The trivial topology is chosen for the pion field, and the QCS is constructed by using the PY projection. We can understand the CPA on the basis of the QCS. We have also shown that the topological difference between the QCS and the hedgehog state is important in the nucleon mass.

Now we comment on a critical issue in the application of the standard coherent state to the baryon physics. As we discussed in this work, the standard coherent state is suitable for the

comprehensive studies of the static pion field in a baryon. As for the excited baryons, the pion spatial excitation must be taken into account in addition to the quark excitation, which is not a serious problem when the ground state properties are considered. In the nonlinear theory of scalar particles, we usually quantize the fluctuation around the static field with minimum energy. However, the isospin symmetry makes this quantization procedure significantly difficult for the standard coherent state. Although the projection method is often employed before proceeding to the spatial quantization, this approach does not actually work because the excitation energy found in this projection is on the same order of the spatial excitation energy [16, 17].

We are now seeking for the general method of constructing a pion state without relying on the standard coherent state in the nonlinear problem. Generalization of the coherent state based on the group theory may be a possible clue to tackle this problem [18].

APPENDIX

The explicit forms of n_k^{ij} $(i,j = \alpha,\beta,\gamma$ and $k = 0,1,2,3)$ in the energy densities are exhibited. For $n_k^{\alpha\alpha}$,

$$\begin{aligned}
n_0^{\alpha\alpha} &= \int \mathrm{d}g\mathrm{d}h F(g,h) \ , && \text{(A.1)}\\
n_1^{\alpha\alpha} &= \sum_{\lambda'\lambda} f_{\lambda'} f_\lambda^* \int dgdh D^1_{00}(h) D^1_{\lambda'\lambda}(g) F(g,h) && \text{(A.2)}\\
n_2^{\alpha\alpha} &= \boldsymbol{f}\cdot\boldsymbol{f}\ n_0^{\alpha\alpha} + n_1^{\alpha\alpha} \ , && \text{(A.3)}\\
n_3^{\alpha\alpha} &= \frac{4}{5}(\boldsymbol{f}\cdot\boldsymbol{f})^2 \left[7n_0^{\alpha\alpha} + 2\int dgdh D^2_{00}(h) F(g,h)\right] + \frac{72}{5}\boldsymbol{f}\cdot\boldsymbol{f} n_1^{\alpha\alpha} \\
&+ 4\sum_{\lambda'\lambda\tau'\tau} f_{\lambda'} f_\lambda^* f_{\tau'} f_\tau^* \int \mathrm{d}g\mathrm{d}h \left[1 + \frac{4}{5} D^2_{00}(h)\right] D^1_{\lambda'\lambda}(g) D^1_{\tau'\tau}(g) F(g,h) \ , && \text{(A.4)}
\end{aligned}$$

where s and $F(g,h)$ are defined in the text.

We can obtain $n_k^{\beta\beta} = n_k^{\gamma\gamma}$ by inserting $1/9 \times D^1_{00}(h) D^1_{00}(g)$ in $n_k^{\alpha\alpha}$. For example,

$$n_0^{\beta\beta} = \frac{1}{9}\int dgdh D^1_{00}(h) D^1_{00}(g) F(g,h) \ . \quad \text{(A.5)}$$

Equations (A.1) and (A.5) are the norms of the QCS $|\boldsymbol{f};00\rangle$ and $|\boldsymbol{f};11\rangle$, respectively.

In the off-diagonal densities, $n^{\alpha\beta} = n^{\alpha\gamma}$, and

$$n^{\alpha\beta} = \frac{1}{9}\left[\boldsymbol{f}\cdot\boldsymbol{f} + \sum_{\sigma} f_\sigma \int dg dh D^1_{00}(h) D^1_{\sigma 0}(g) F(g,h)\right] . \tag{A.6}$$

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